# Quantum corrections to Hawking radiation spectrum

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## Abstract

In 1995, Bekenstein and Mukhanov suggested that the Hawking radiation spectrum would be discrete if the area spectrum is quantized in such a way that the allowed area is the integer multiples of a single unit area. However, in 1996, Barreira, Carfora and Rovelli argued that the Hawking radiation spectrum is continuous if the area spectrum is quantized in such a way that there are not only a single unit area as Bekenstein and Mukhanov naively considered, but also multiple numbers, actually infinite numbers, of unit area as loop quantum gravity predicts. In this paper, contrary to what they argued, by adding a further hypothesis of the locality of photon emission in black hole, we show that the Hawking radiation spectrum is still discrete even in the case that the allowed area is not simply the integer multiples of a single unit area, as long as the area spectrum is quantized as loop quantum gravity predicts. In particular, we show that the Hawking radiation spectrum is truncated below a certain frequency, given a certain temperature of black hole.

# 1  Introduction

By now, it is very well-known that a black hole must emit particles, thanks to Hawking [1]. He also argued that the radiation spectrum must follow the Planck radiation spectrum. However, strictly speaking, this may not be the case, as Hawking's calculation was semi-classical rather than full-quantum. Perhaps, from these considerations, Bekenstein and Mukhanov showed that the Hawking radiation spectrum would be discrete if the allowed area is the integer multiples of unit area [2]. In this paper, by assuming the locality of photon emissions from black hole (i.e. a photon is emitted from a single area quantum in the black hole, rather than from extended regions), which leads to the selection rule for quantum black hole, we argue that the Hawking radiation spectrum is still discrete contrary to what Barreira, Carfora and Rovelli argued [3] and Krasnov argued [4], even in the case that the allowed area is not simply the integer multiples of unit area, as long as the area spectrum is quantized as loop quantum gravity predicts [5, 6, 7].

In section 2, we will closely explain the motivations behind our hypothesis and show that that leads to the selection rule for quantum black holes, namely, that the area of black hole can decrease only by the amount equal to the area spectrum, upon emission of a photon. In section 3, by relating this area deduction with the energy of photon emitted, we will show that this selection rule leads to the discreteness of Hawking radiation spectrum. Also, in that section, we will show that the Hawking radiation spectrum is truncated below certain energy of photon. To this end, we will consider three different area spectrums in the market: First, isolated horizon framework [8, 9]. Second, the one by Tanaka and Tamaki [10]. Third, the one by Kong and Yoon [11, 12, 13]. In all cases, we find that the Hawking radiation is discrete. In section 4, we present an alternative, but equivalent derivation for the relation between the area deduction and the energy of photon emitted, which again leads to this discreteness. In section 5, we present how our result will change if we consider logarithmic corrections to the Bekenstein-Hawking entropy. In section 6, we conclude our article.

# 2  Selection rules for quantum black holes

In their article [3], Barreira, Carfora and Rovelli noticed that "the spacing of the energy levels [of black hole] decreases *exponentially* with M." They went on to say, "It follows that for a macroscopical black hole the spacing between energy levels is infinitesimal, and thus the spectral lines are virtually dense in frequency." Their argument that for a macroscopical black hole the spacing between energy levels is infinitesimal is correct. However, their argument that the spectral lines are virtually dense in frequency can be

controversial, if we add a further hypothesis. They thought that a photon emitted from a black hole can have any energy provided that this energy happens to be the difference of any two energies an arbitrary black hole can have. Therefore, they implicitly assumed that a black hole can turn into any other black hole with any arbitrary energy provided that the black hole that it turns to has less energy than before. Similarly, they implicitly assumed that a black hole can turn into any other black hole with any arbitrary area provided that the black hole that it turns to has less area than before.

To ease understanding, let me phrase what they thought in mathematical formulas and explain why it is controversial if we add a hypothesis. Let's say that we have the following area eigenvalues:

$$A_{spec} = A_1, A_2, A_3, A_4, A_5, A_6.... \tag{1}$$

Then, the black hole area $A$ must be given by the following formula.

$$A = \sum_j N^j A_j \tag{2}$$

where $N^j$s are non-negative integers. Here, we can regard the black hole as having $\sum N^j$ partitions, each of which has the area $A_{spec}$. In this mathematical language, we can express the consideration of Barreira, Carfora and Rovelli as follows: They thought that the black hole with the initial area $A_i = \sum N_i^j A_j$ can turn into the final area $A_f = \sum N_f^j A_j$ with the emission of photons, as long as $A_f$ is less than $A_i$, without any restrictions to $N_f^j$.

However, if we assume that the emission of a photon should be local, this is not the case. In other words, a photon should be emitted from a single area quantum of the black hole, rather than from scattered or extended regions of the black hole; one cannot imagine that a single photon could be emitted from two or more places simultaneously. Perhaps from these considerations, Krasnov argued as followings [4]:

"Consider a quantum process in which the black hole jumps from a state $|\Gamma >$ to state $|\Gamma' >$, such that the horizon area changes. This, for example, can be a process in which one of the flux lines piercing the horizon breaks, with one of the ends falling into the black hole and the other escaping to infinity (see Fig. 1b). This is an example of the emission process; the two ends of the flux line can be thought of as the two particle anti-particle quanta in Hawkings original picture [6] of the black hole evaporation"

Translating this into a mathematical formula, what Krasnov argues is the following:

$$\Delta A = A_j - A_i \tag{3}$$

for some $A_i$ bigger than $A_j$. In other words, the partition with the area $A_i$ on the black

hole horizon is shrunken into the partition with the area $A_j$ upon the emission of a particle, because of the anti-particle reaching this partition of the black hole horizon.

However, this is troublesome from the following reasons. The observer near the black hole horizon must observe that the energy of pair-created particle and anti-particle should be the same. This consideration may impose some conditions on how much the energy must be divided between each of particle and anti-particle that are pair-created. This consideration is lacking in the above article; Krasnov regarded the situation as if any proportion of energy can be divided between the particle and anti-particle upon Hawking radiation. If this is considered, there should be some more conditions on the selection rule, or equivalently the state $|\Gamma' >$ to which $|\Gamma >$ can transform into.

Given this, let's step further. Let's say a single flux line breaks into a particle and an anti-particle and the particle reaches us who are very far from the black hole. In this case, we will never see that the anti-particle reaching the black hole as the time is infinitely dilated, even though we see that particle reaches us. In other words, we will never see the anti-particle, which should transform $|\Gamma >$ into $|\Gamma' >$ upon the absorption by black hole, be absorbed. In conclusion, the whole single flux line must be completely vanished from the black hole at the first place, as we never see the effect of anti-particle. In other words, a single flux line never breaks into two. The whole single flux line emitted reaches us. From these considerations, I propose the following equation.

$$\Delta A = -A_i \tag{4}$$

for some $i$. Compare the above formula with (3). As whole single flux line is emitted, there is no remnant from this flux line on the black hole horizon. Therefore, we set $A_j = 0$ in (3).

The above equation will turn out to be crucial to our derivation of the discreteness of the Hawking radiation spectrum in the next section.

## 3   The discreteness of the Hawking radiation spectrum

From black hole thermodynamics, we know the following, as in [1]:

$$r = 2M \tag{5}$$

$$A = 4\pi r^2 = 16\pi M^2 \tag{6}$$

$$kT = \frac{1}{8\pi M} \tag{7}$$

Here, $A$ is the horizon area of the black hole, $T$ its temperature, $r$ its radius, and $M$ its mass. For simplicity we considered Schwarzschild black hole case, as one can easily generalize it to a generic black hole as in section 4. Now, consider the case when a

photon is emitted. As a photon is emitted, the black hole loses energy, and thus its area decreases by a certain amount, which must be $A_{spec}$, the area spectrum predicted by loop quantum gravity, as we argued in the last section. From this consideration, we can calculate $E_{photon}$, the energy of the emitted photon as follows.

First of all, the mass of the black hole decreases as follows.

$$\Delta M = -E_{photon} \tag{8}$$

Then, considering (6),(7) the area of the black hole decreases as follows.

$$\Delta A = 32\pi M \Delta M = -32\pi M E_{photon} = -\frac{4E_{photon}}{kT} = -A_{spec} \tag{9}$$

where in the last step we used the fact that the black hole area must be decreased by $A_{spec}$ the area spectrum allowed by loop quantum gravity. Therefore, we conclude the following:

$$E_{photon} = \frac{A_{spec}}{4} kT \tag{10}$$

Here, we see easily that the energy of the emitted photon is quantized, as $A_{spec}$ is quantized. In particular, as loop quantum gravity predicts that there is non-zero minimum area, there is non-zero energy for the photons emitted given the temperature of a black hole.

In case of isolated horizon framework [8, 9], the minimum area is given by $4\pi\sqrt{3}\gamma$ where $\gamma$ is the Immirzi parameter. Therefore, we have the following for the minimum energy of emitted photon.

$$E_{min} \approx 1.49kT \tag{11}$$

In case of Tanaka-Tamaki scenario [10], the minimum area is given by $4\pi\gamma$ where $\gamma$ is the Immirzi parameter in this case. This gives the following for the minimum energy of emitted photon.

$$E_{min} \approx 2.462kT \tag{12}$$

In case of Kong-Yoon scenario [11, 12, 13], the minimum area is given by $4\pi\sqrt{2}$. Therefore, we have the following.

$$E_{min} \approx 4.44kT \tag{13}$$

Therefore, the Hawking radiation is truncated below this energy. See the figures. The discrete Hawking radiation spectrums are represented by solid lines.
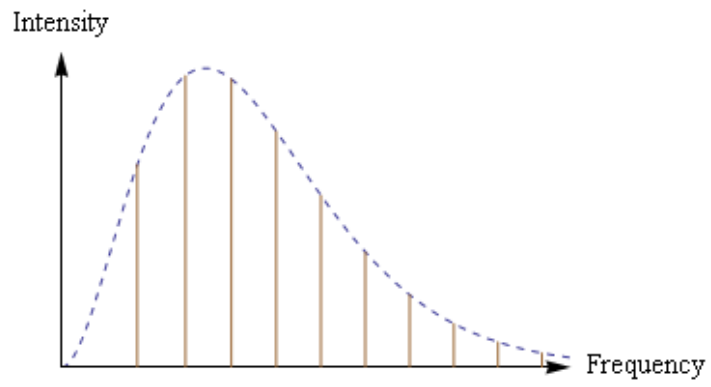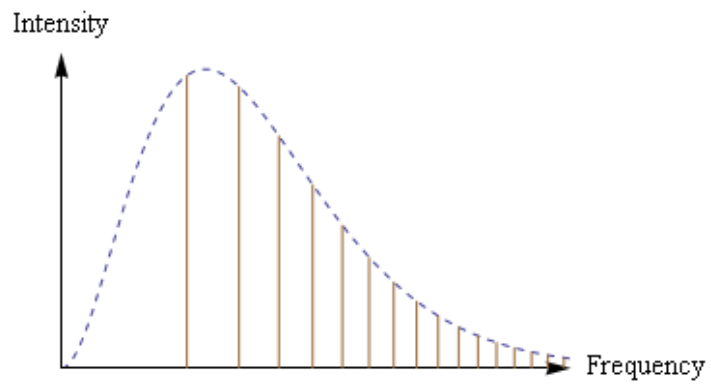
Figure 1: Isolated horizon framework


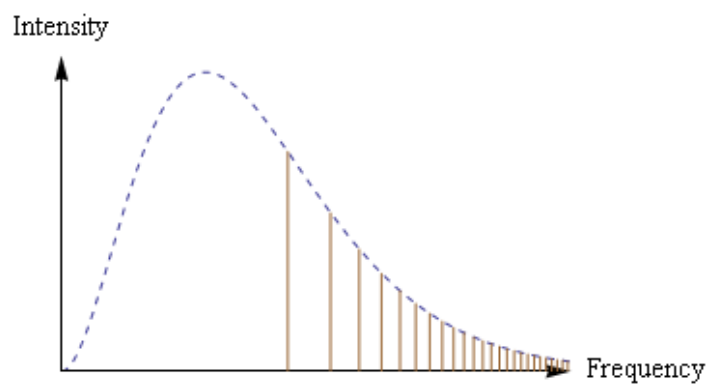
Figure 2: Tanaka-Tamaki scenario



Figure 3: Kong-Yoon scenario

## 4    Alternative derivation

In this section, we present a simpler derivation. From thermodynamics, we have the following:

$$\Delta Q = T \Delta S \tag{14}$$

Plugging the followings,

$$\Delta Q = -E_{photon} \tag{15}$$

$$\Delta S = -kA_{spec}/4 \tag{16}$$

we recover (10).

## 5    The logarithmic corrections to the black hole entropy

In the presence of the logarithmic corrections to the black hole entropy, (10) is modified. In this section, we consider the fully $SU(2)$ framework as an example. Other cases can be dealt in a similar way. In the fully $SU(2)$ framework, we have[14, 15, 16, 17]:

$$S = \frac{A}{4} - \frac{3}{2} \ln A + O(1) \tag{17}$$

Given this, using (14),(15),(6) and (7), we obtain:

$$E_{photon} = (\frac{kT}{4} - 6\pi (kT)^3) A_{spec} \tag{18}$$

## 6    Discussion and Conclusions

In this article, by proposing a selection rule for quantum black holes from a hypothesis of the locality of the photon emission, we showed that the Hawking radiation spectrum must be discrete, and in particular, truncated below certain frequency, deviating from the simple Planck radiation spectrum. It may be possible that this phenomenon is confirmed, if the Hawking radiation is observed at LHC.

## 7    Acknowledgement

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